Combined 1st & 2nd Sem BTech Degree Exam, April 2015

EN 14 105 : Engineering Mechanics

Answer Scheme for Model Question Paper.

Part A (Answer Eight Out of Ten) (5 Marks)

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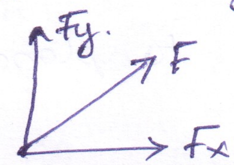
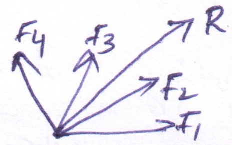
(1) Differentiate Resolution and Composition of Forces ?

Resolution is splitting up of forces into its rectangular components or it is the process of finding number of component forces which will have the same effect as the given single forces.

Composition is exactly opposite to the Resolution. It is the process of combining different forces acting at different directions to get a single force known as resolution

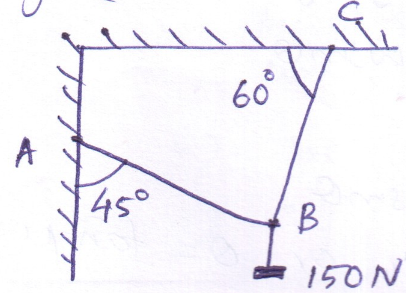


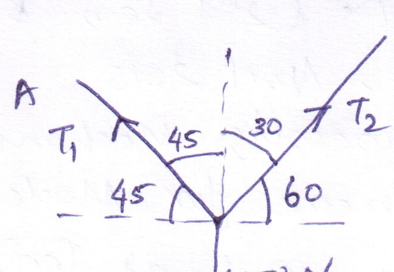
Resolution



composition.

(2) ~~Find~~ forces developed in the wires, supporting an electric fixture as shown in Fig (1).



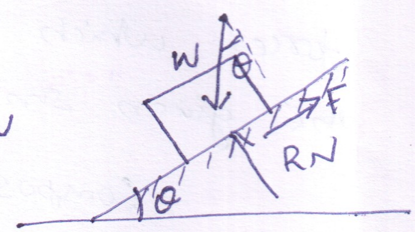


Applying Lami's Theorem,

$$\frac{T_1}{\sin(90+60)} = \frac{T_2}{\sin(90+45)} = \frac{150}{\sin(45+30)}$$

$T_1 = 77.6\,N$ & $T_2 = 109.8\,N$.

③ Determine the slope of an inclined plane, above which a weight will slide down. The coefficient of friction is μ.



A block of weight W resting on inclined plane which makes an angle θ with horizontal at which the block starts sliding down the plane.

Resolving along normal to plane,

$R_N - W\cos\theta = 0$

$R_N = W\cos\theta$ ——— (I)

Resolving along $\parallel^{lel}$ to plane,

$F - W\sin\theta = 0.$

$F = W\sin\theta.$ ——— (II)

We know limiting friction, $F = \mu R_N$.

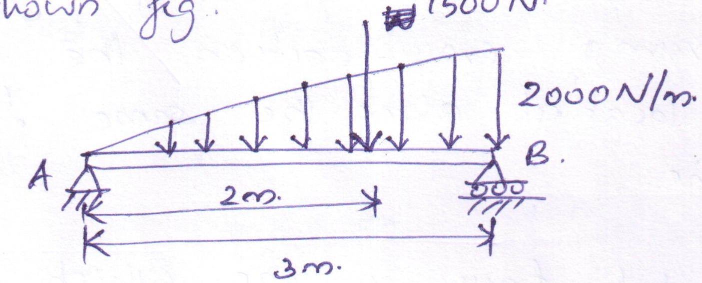
$\therefore \mu R_N = W\sin\theta$

Sub, (I) in (II),

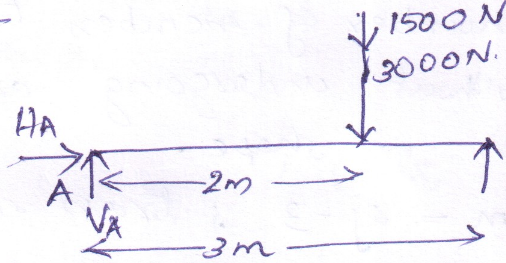
$\mu . W\cos\theta = W\sin\theta$

$\tan\theta = \mu$ or $\theta = \tan^{-1}\mu = \alpha$, angle of repose

1) Find reactions at the supports of the beam as shown fig.



Ans :



$\frac{1}{2} WL$

$= \frac{1}{2} \times 2000 \times 3$

$= \underline{3000 N}$

@ $2/3 \times 3 = 2m$ from A.

$\Sigma M_A = 0.$

$R_B \times 3 - 4500 \times 2 = 0.$

$R_B = \underline{3000 N}$

$\Sigma M_B = 0.$

$H_A \times 0 + V_A \times 3 - 4500 \times 1 = 0$

$V_A = \underline{1500 N}$

$\Sigma H = 0.$

$H_A = 0.$

$\therefore R_A = V_A = 1500 N ; \quad R_B = 3000 N$

5) i) List out assumptions which are made in the analysis of truss.

ii) Differentiate between perfect truss and deficient truss.

Ans: i) a) The ends of the members are pin-joined (hinged)

b) The loads act only at joints

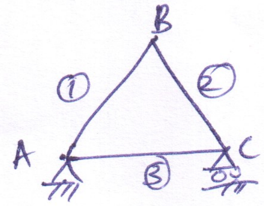
c) Self weight of members are negligible

iii) Members are having either uniform cross-sections throughout if they have varying cross-section the centroid is located along the same longitudinal line.

ii) Perfect truss is one which has got sufficient number of members to resist the loads without undergoing appreciable deformation in shape.

ie $m = 2j - 3$; truss satisfies the equation, where m = no: of members, j - no: of joints

eg.



$m = 3$

$j = 3$

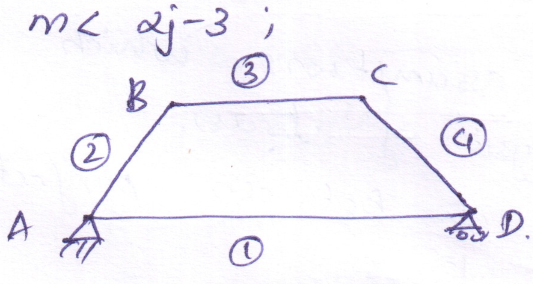
$2j - 3 = 2 \times 3 - 3 = \underline{\underline{3}}$

ie $\underline{\underline{m = 2j - 3}}$

Deficient truss is one which has less number of members to resist the loads without undergoing appreciable deformation in shape.

ie $m < 2j - 3$;

eg.



$m = 4$

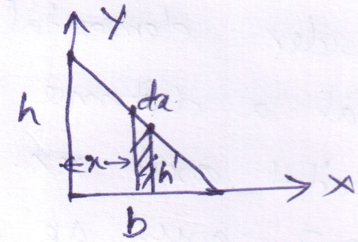
$j = 4$.

$2j - 3 = 2 \times 4 - 3$

$= \underline{\underline{5}}$

$\underline{\underline{m < 2j - 3}}$

6) Find centroid of a right angled triangle of base 'b' and height 'h'.



$$dA = h' \times dx$$

$$= \frac{h}{b}(b-x)\, dx$$

$$\frac{h'}{h} = \frac{b-x}{b}.$$

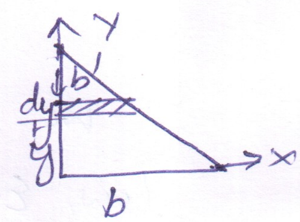
$$h' = \frac{b-x}{b} \times h$$

$$\bar{x} = \frac{\int x\, dA}{\int dA}$$

$$= \frac{\int_0^b \frac{h}{b}(b-x)\, x\, dx}{\int_0^b \frac{h}{b}(b-x)\, dx}$$

$$= \frac{\frac{h}{b} \cdot \left. \frac{bx^2}{2} + \frac{x^3}{3} \right]_0^b}{\left. \frac{h}{b}\; bx - \frac{x^2}{2} \right]_0^b}$$

$$= \frac{\left(\frac{b^3}{2} + \frac{b^3}{3}\right)}{\left(b^2 - \frac{b^2}{2}\right)} = \frac{b^3/6}{b^2/2} = \underline{\underline{b/3}}$$



$$dA = b' \times dy.$$

$$\bar{y} = \frac{\int y\, dA}{\int dA}$$

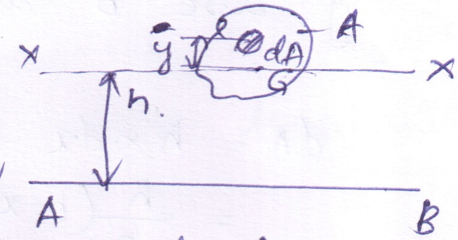
$$= \frac{\int \frac{b}{h}(h-y)\, y\, dy}{\int \frac{b}{h}(h-y)\, dy}$$

$$\frac{b'}{b} = \frac{(h-y)}{h}.$$

$$b' = \frac{b}{h}(h-y)$$

$$= \frac{\left. h\frac{y^2}{2} + \frac{y^3}{3} \right)_0^h}{\left. hy - \frac{y^2}{2} \right]_0^h} = \frac{h^3/6}{h^2/2} = \underline{\underline{\frac{h}{3}}}$$

7) State and Prove Parallel axis theorem.



consider elemental area dA at a distance $\bar{y}$ from centroidal axis XX. Consider another axis AB at distance h from XX axis.

Moment of inertia of dA about AB

$$= (h+\bar{y})^2 \times dA$$

$\therefore$ Total MI abt AB $= \int dA\,(h+y)^2$

$$= \int (h^2+2hy+y^2)\,dA$$

$$= \int h^2\,dA + \int 2hy\,dA + \int y^2 dA$$

$$= h^2 . A + 2h . \int y\,dA + I_{xx}$$

$$= \underline{\underline{I_{xx} + Ah^2}}$$

$\because \int y\,dA = \bar{y}\,A = 0$ (since y is taken from XX axis)

8) Derive Work - Energy Principle.

We know that, as per Newton's second law, $\Sigma F = ma$.

$$\Sigma F = m . \frac{dv}{dt} = m . \frac{dv}{ds} . \frac{ds}{dt}.$$

$$\cancel{\Sigma F . dt = m dv.}$$

integrating, $\cancel{\int_0^t \Sigma F . dt = \int_0^v m dv}$

$\cancel{\Sigma F . t\Big]_0^t = m . v\Big]_0^v}$

$$\Sigma F = m . v\frac{dv}{ds}.$$

$$\int_{s=0}^{s} \Sigma F \times ds = \int_{v=u}^{v} m v\,dv \quad \text{ie;} \quad \Sigma F . (S-0) = m . \frac{v^2}{2}\Big]_u^v$$

$$\Sigma F \times S = \frac{1}{2} m v^2 - \frac{1}{2} m u^2$$

A rigidbody rotates about a fixed axis and slows down from 300rpm to 150rpm in 2 minutes. Determine angular acceleration and the number of revolutions completed in 2 minutes.

$$\omega = \omega_0 + \alpha t$$

$$150 = 300 + \alpha \times 2$$

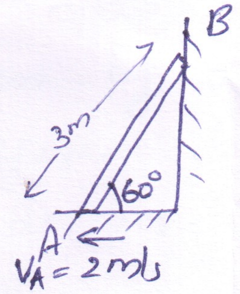
$$\alpha = \cancel{2}\ \frac{150-300}{2} = \frac{150}{2} = \underline{75 \text{ rpm/minute}}$$

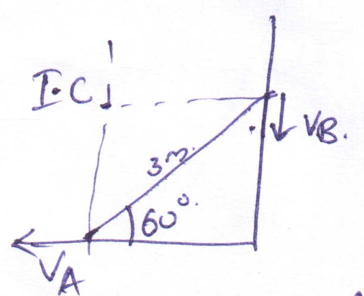
$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$= 300 \times 2 + \frac{1}{2} \times 75 \times 2^2$$

$$= 600 - 150 = \underline{450 \text{ revolutions}}$$

10) Find the velocity of B of the rod as shown fig.





$$V_A = AC \times \omega.$$

$$2 = 3\sin 60 \times \omega$$

$$\omega = \underline{0.770} \text{ rad/sec}$$

$$V_B = BC \times \omega$$

$$= 3\cos 60 \times 0.770$$

$$= \underline{1.555 \text{ m/sec}}$$

PART B

1) **Calculate the resultant moment about the corner *B* shown in fig. 5.32.**

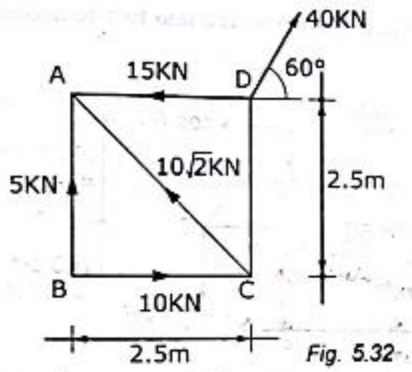


Fig. 5.32

Solution:

Inclined forces 40KN and $10\sqrt{2}\ KN$ are resolved into two components and the given system of force is taken as shown in fig. 5.33.

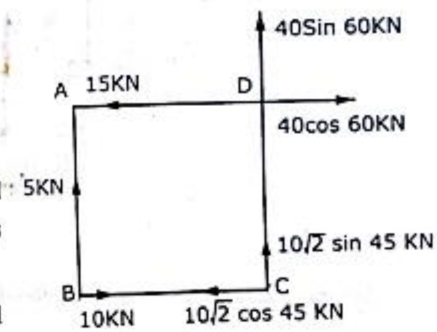


Fig. 5.33.

For resultant moment at *B*, Take the moment of all the forces about *B* and find the algebraic sum.

∴ Resultant moment at *B*,

$$\Sigma M_B = (5 \times 0) + (10 \times 0) - (10\sqrt{2}\cos 45 \times 0) - (15 \times 2.5) - (10\sqrt{2}\sin 45 \times 2.5) - (40\sin 60 \times 2.5) + (40\cos 60 \times 2.5)$$

$$= -37.5 - 25 - 86.6 + 50$$

$$= -99.1\ KNm \quad \text{('–' shows anticlockwise)}$$

∴ Resultant moment at $B = 99.1\ KNm$ (anticlockwise)

OR

2 ***A simply supported overhanging beam 20m long carries a system of loads and a couple as shown in fig 8.23. Determine the reactions at supports A and B.***

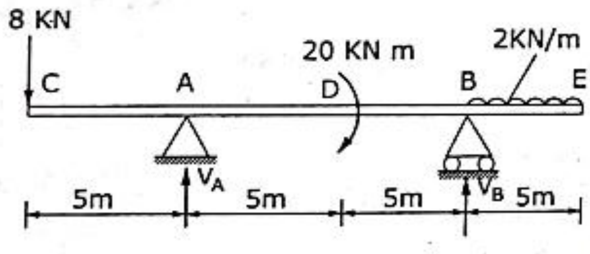


Fig. 8.23.

Solution.

Applying $\Sigma H = 0$

As three is no horizontal force on the beam, $H_A = 0$

Applying $\Sigma V = 0$

$$V_A + V_B - 8 - (2 \times 5) = 0$$

$$\therefore V_A + V_B = 18 \qquad \text{--- (i)}$$

Applying $\Sigma M_A = 0$

$$(2 \times 5 \times 12.5) - (8 \times 5) + 20 - (V_B \times 10) = 0$$

Note :

Couple of 20*KNm*, is not to be considered for $\Sigma H = 0$ and $\Sigma V = 0$ equations. In $\Sigma M_A = 0$ equation it takes positive measure, because, given couple is clockwise.)

Solving we get $V_B = 10.5\ KN$

Substituting $V_B = 10.5 KN$ in equation (i), we get,

$V_A = 7.5\ KN$

Result : $H_A = 0$; $V_A = 7.5\ KN\ (\uparrow)$; $V_B = 10.5\ KN\ (\uparrow)$

3 **Find the moment of inertia of plane area shown below in fig. 13.40 about its centroidal axes.**

Solution

Location of Centroid

The given lamina is divided into 3 portions

Portion (1) : (Rectangle)

area, $a_1 = 8 \times 12 = 96 \text{ cm}^2$

$$x_1 = (12 - 8) + \frac{8}{2} = 8 \text{ cm}$$

$$y_1 = \frac{12}{2} = 6 \text{ cm}$$

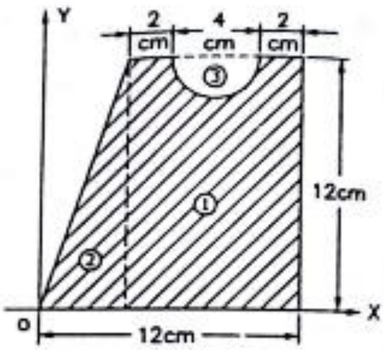


Fig. 13.40

Portion (2) : (Triangle)

area, $a_2 = \frac{1}{2} \times (12 - 8) \times 12 = 24 \text{ cm}^2$

$$x_2 = \frac{2}{3} \times (12 - 8) = 2.67 \text{ cm}$$

$$y_2 = \frac{12}{3} = 4 \text{ cm}$$

Portion (3) : (Semi-circle)

area, $a_3 = \frac{1}{2} \times \frac{\pi \times 4^2}{4} = 6.283 \text{ cm}^2$

$$x_3 = (12 - 8) + 2 + \frac{4}{2} = 8 \text{ cm}$$

$$y_3 = 12 - \left(\frac{4 \times 2}{3\pi}\right) = 11.15 \text{ cm}$$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 - a_3 x_3}{a_1 + a_2 - a_3}$$

$$= \frac{(96 \times 8) + (24 \times 2.67) - (6.283 \times 8)}{96 + 24 - 6.283}$$

$$= 6.875 \text{ cm}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 - a_3 y_3}{a_1 + a_2 - a_3}$$

$$= \frac{(96 \times 6) + (24 \times 4) - (6.283 \times 11.15)}{96 + 24 - 6.283}$$

$$= 5.293 \text{ cm}$$

The centroidal axes of the given lamina are drawn as below in fig. 13.41.

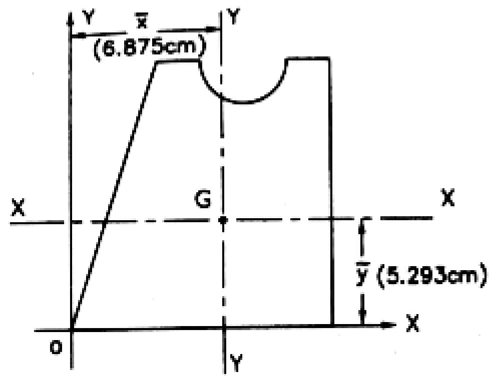


Fig. 13.41.

Moment of Inertia

Moment of Inertia about horizontal centroidal axis (I_{xx})

$$I_{xx} = I_1 + I_2 - I_3$$

where $I_1 = I_{G1} + A_1\bar{h}_1^2$

$$= \left(\frac{8 \times 12^3}{12}\right) + [(8 \times 12) \times 0.767^2]$$

$$= 1199.98 \text{ cm}^4$$

$$\bar{h}_1 = \bar{y} \sim y_1 = 5.293 \sim 6 = 0.707 \text{ cm}$$

$$I_2 = I_{G2} + A_2\bar{h}_2^2$$

$$= \frac{4 \times 12^3}{36} + \left[\left(\frac{1}{2} \times 4 \times 12\right) \times 1.293^2\right]$$

$$= 232.12 \text{ cm}^4$$

$$\bar{h}_2 = \bar{y} \sim y_2 = 5.293 \sim 4 = 1.293 \text{ cm}$$

$$I_3 = I_{G3} + A_3\bar{h}_3^2$$

$$= \left(0.11 \times 2^4\right) + \left[\left(\frac{1}{2} \times \frac{\pi \times 4^2}{4}\right) \times (5.858)^2\right]$$

$$= 217.37 \text{ cm}^4$$

$$\bar{h}_3 = \bar{y} \sim y_3 = 6.707 - \left(\frac{4 \times 2}{3\pi}\right) = 5.858 \text{ cm}$$

$$\therefore I_{xx} = I_1 + I_2 - I_3$$

$$= 1199.98 + 232.12 - 217.37$$

$$= 1214.73 \text{ cm}^4$$

Moment of Inertia about vertical centroidal axis (I_{yy})

$$I_{yy} = I_1 + I_2 - I_3$$

where $I_1 = I_{G1} + A_1\bar{h}_1^2$

$$= \left(\frac{12 \times 8^3}{12}\right) + [(12 \times 8) \times 1.125^2]$$

$$= 633.5 \text{ cm}^4$$

$\bar{h}_1 = \bar{x} \sim x_1 = (6.875 \sim 8) = 1.125$ cm

$$I_2 = I_{G2} + A_2\bar{h}_2^2$$

$$= \left(\frac{12 \times 4^3}{36}\right) + \left[\left(\frac{1}{2} \times 12 \times 4\right) \times 4.205^2\right]$$

$$= 445.70 \text{ cm}^4$$

$\bar{h}_2 = \bar{x} \sim x_2 = (6.875 \sim 2.67) = 4.205$ cm

$$I_3 = I_{G3} + A_3\bar{h}_3^2$$

$$= \left(\frac{1}{2} \times \frac{\pi \times 4^4}{64}\right) + \left[\left(\frac{1}{2} \times \frac{\pi \times 4^2}{4}\right) \times 1.125^2\right]$$

$$= 14.235 \text{ cm}^4$$

$\bar{h}_3 = \bar{x} \sim x_3 = (6.875 \sim 8) = 1.125$ cm

$$\therefore \quad I_{yy} = I_1 + I_2 - I_3$$

$$= 633.5 + 445.7 - 14.235 = 1064.96 \text{cm}^4$$

$$\boxed{\begin{aligned} \therefore \quad \mathbf{I_{xx}} &= \mathbf{1214.73 \text{ cm}^4} \\ \text{and } \mathbf{I_{yy}} &= \mathbf{1064.96 \text{cm}^4} \end{aligned}}$$

OR

4 *A truss supported on an inclined plane at an angle of 30° with horizontal is shown in fig 9.68. Determine the forces in all the members.*

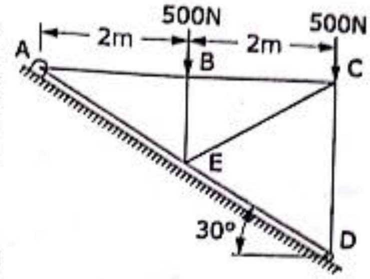


Fig 9.68

Solution

Support Reactions:

In Fig 9.72, the truss is roller supported at D and Hinged at A. The assumed directions of support reactions are shown below (Fig. 9.69)

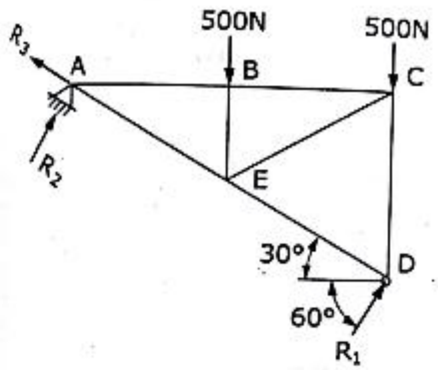


Fig 9.69

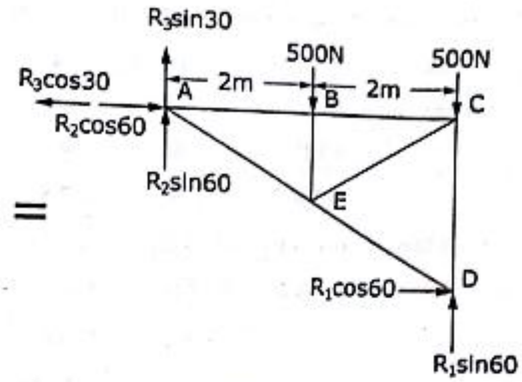


Fig 9.70

The reactions R_1, R_2 and R_3 are resolved into components along x and y directions as shown in fig 9.70

Applying $\Sigma \mathbf{H} = \mathbf{0}$ $(\rightarrow +)$

$$R_1 \cos 60 + R_2 \cos 60 - R_3 \cos 30 = 0 \qquad \text{............ (i)}$$

Applying $\Sigma \mathbf{V} = \mathbf{0}$ $(\uparrow +)$

$$R_1 \sin 60 + R_2 \sin 60 + R_3 \sin 30 - 500 - 500 = 0$$

or $$R_1 \sin 60 + R_2 \sin 60 + R_3 \sin 30 = 1000 \qquad \text{............ (ii)}$$

Applying $\Sigma \mathbf{M_A} = \mathbf{0}$ $(\circlearrowright +)$

$$(500 \times 2) + (500 \times 4) - (R_1 \cos 60 \times CD) - (R_1 \sin 60 \times AC) = 0$$

From the geometry of the truss,

$$\tan 30 = \frac{CD}{AC} \qquad \therefore CD = AC \tan 30$$

$$AC = 4\text{m} \qquad = 4 \tan 30$$

$$= 2.309 \text{ m}$$

$\therefore\ (500 \times 2) + (500 \times 4) - (0.5\, R_1 \times 2.309) - (0.866\, R_1 \times 4) = 0$

$$1000 + 2000 - 1.1545\, R_1 - 3.464\, R_1 = 0$$

$$3000 = 4.6185\, R_1$$

$$\therefore\ R_1 = 649.5 \text{ N}$$

Substitute R_1 in eqn (i)

$$649.5 \cos 60 + 0.5\, R_2 - 0.866\, R_3 = 0$$

or $\quad 0.5\, R_2 - 0.866\, R_3 = 324.75$ .... (iii)

$|||^{ly}$ Substitute R_1 in eqn (ii)

$$649.5 \sin 60 + R_2 \sin 60 + R_3 \sin 30 = 1000$$

or $\quad 0.866\, R_2 + 0.5\, R_3 = 437.5$ .... (iv)

Solve the equations (iii) & (iv)

$$0.5\, R_2 - 0.866\, R_3 = 324.75$$

$$0.866\, R_2 + 0.5\, R_3 = 437.5$$

(iii) $\times 0.866 \Rightarrow 0.433\, R_2 - 0.75\, R_3 = 281.23$

(iv) $\times 0.5 \Rightarrow 0.433\, R_2 + 0.25\, R_3 = 218.75$

$$-R_3 = 62.48$$

$$\therefore\ R_3 = -62.48$$

Substitute R_3 in eqn (iii)

$$0.5\, R_2 - 0.866\, R_3 = 324.75$$

or $\quad 0.5\, R_2 - (0.866 \times -62.48) = 324.75$

or $\quad 0.5\, R_2 = 270.64$

$$\therefore\ R_2 = 541.28 \text{ N}$$

The Reactions are shown in fig 9.71

(Note: R_1 and R_2 are positive hence their assumed directions are correct, but R_3 is negative, so the components of R_3 are taken in the opposite directions as shown in Fig 9.71)

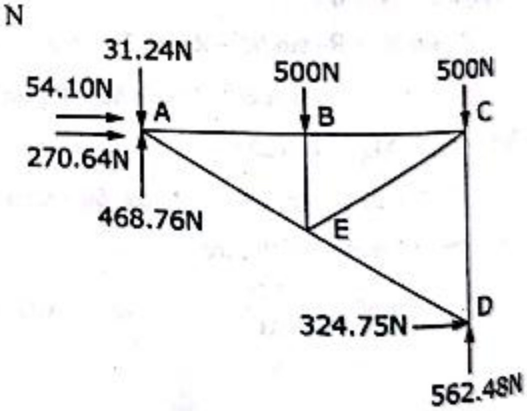


Fig 9.71

Member Forces.

Joint D (Fig 9.72)

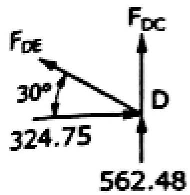


Fig 9.72

$\Sigma H = 0$

$$324.75 - F_{DE} \cos 30 = 0$$

$$\therefore \quad F_{DE} = 374.98 \text{ N (Tensile)}$$

$\Sigma V = 0$

$$F_{DC} + F_{DE} \sin 30 + 562.48 = 0$$

$$F_{DC} + 374.98 \sin 30 + 562.48 = 0$$

$$\therefore \quad F_{DC} = -750 \text{ N (Comp)}$$

Joint C (Fig 9.73)

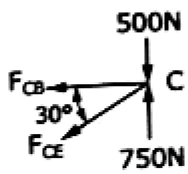


Fig 9.73

$\Sigma V = 0$

$$F_{CE} \sin 30 - 500 + 750 = 0$$

$$\therefore \quad F_{CE} = -500 \text{ N (Comp)}$$

$\Sigma H = 0$

$$-F_{CB} - F_{CE} \cos 30 = 0$$

$$-F_{CB} - (-500 \cos 30) = 0$$

$$-F_{CB} + 433 = 0$$

$$\therefore \quad F_{CB} = 433 \text{ N (Tensile)}$$

Joint B (Fig 9.74)

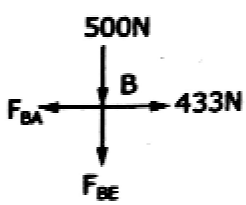


Fig 9.74

$\Sigma V = 0$

$$-500 - F_{BE} = 0$$

$$\therefore \quad F_{BE} = -500 \text{ N (Comp)}$$

$\Sigma H = 0$

$$-F_{BA} + 433 = 0$$

$$\therefore \quad F_{BA} = 433 \text{ N (Tensile)}$$

Joint A (Fig 9.75)

$\Sigma V = 0$

$$-31.24 - F_{AE} \sin 30 + 468.76 = 0$$

$$\therefore \quad F_{AE} = 875 \text{ N (Tensile)}$$

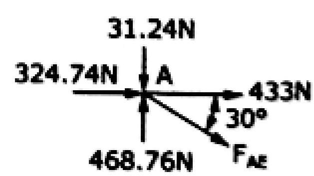
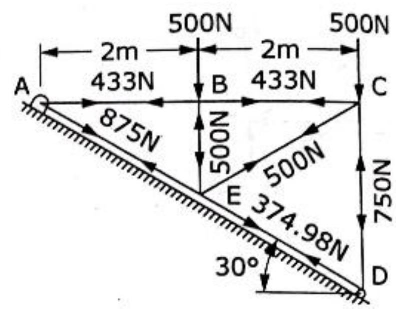


Fig 9.75

The results are shown in Fig 9.76 and Table 9.10

Table 9.10

Sl.No.	Member	Force in N	Nature
1	AB	433	Tensile
2	BC	433	Tensile
3	CD	750	Compressive
4	DE	374.98	Tensile
5	EA	875	Tensile
6	BE	500	Compressive
7	EC	500	Compressive



5 *The angle of rotation of a body is given by the equation,*
$\theta = 3t^3 + 4t^2 - 6t + 9$, *where* θ *is expressed in radians and t in seconds, find :*
(i) angular velocity and
(ii) angular acceleration of the body when $t = 0$ *and* $t = 4$ *sec*

Solution :

Given, $\theta = 3t^3 + 4t^2 - 6t + 9$

$$\therefore \text{angular velocity, } \omega = \frac{d\theta}{dt}$$
$$= \frac{d}{dt}(3t^3 + 4t^2 - 6t + 9)$$
$$= (9t^2 + 8t - 6) \quad \text{........ (i)}$$

$$\text{angular acceleration, } \alpha = \frac{d\omega}{dt}$$
$$= \frac{d}{dt}(9t^2 + 8t - 6)$$
$$= 18t + 8 \quad \text{........ (ii)}$$

(i) Angular velocity

substitute the values t in equation (i) to find the corresponding angular velocity

when $t = 0$: $\omega_o = -6$ rad/s

when $t = 4$ sec, $\omega_4 = (9 \times 4^2) + (8 \times 4) - 6$

$= 170$ rad/s

(ii) Angular acceleration

Substitute the values of 't' in equation (ii) to find the corresponding angular acceleration

when $t = 0$; $\alpha_o = 8 \text{ rad / s}^2$

when $t = 4$ sec : $\alpha_4 = (18 \times 4) + 8$

$= 80 \text{ rad/s}^2$

OR

6 **A Bullet of mass 30 gm is fired horizontally into a body of mass 10 Kg, which is suspended by a string of 0.8m long. Due to this impact, the body swings through an angle of 30°. Find the velocity of the Bullet.**

Solution:

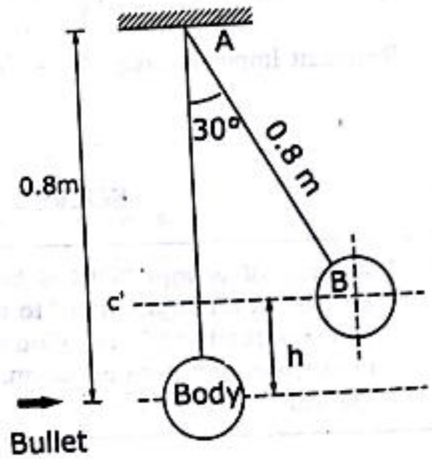


Fig. 20.24

Given, mass of bullet, $m_1 = 0.03$ Kg

mass of body, $m_2 = 10$ Kg

length of string, $l = 0.8$m

angle of swing, $\theta = 30°$

Find the velocity of bullet?

Let the velocity of bullet, before impact be u_1

The velocity of body, before impact, $u_2 = 0$ ($\because$ at rest)

Total mass of the system = $M = (m_1 + m_2)$

$= (0.03 + 10) = 10.03$ Kg

Let, the final velocity of the system after impact be V

Applying law of conservation of momentum,

Total momentum before impact = Total momentum after impact.

i.e., $(m_1 \times u_1) + (m_2 \times u_2) = MV$

or $0.03u_1 + (10 \times 0) = 10.03\ V$

or $0.03\ u_1 = 10.03\ V$ ..... (i)

Let the body is raised to a height, h after impact,

From the law of conservation of energy,

Potential energy of the system at B = Kinetic energy of the system after impact.

$$\text{i.e.,} \quad Mgh = \frac{1}{2} MV^2 \qquad \text{but, } h = 0.8 - 0.8 \cos 30 = 0.107 \text{ m}$$

$$10.03 \times 9.81 \times 0.107 = \frac{1}{2} \times 10.03 \times V^2$$

$$\text{or} \quad 10.528 = 5.015 \, V^2$$

$$\therefore \quad V = \sqrt{\frac{10.528}{5.015}} = 1.448 \text{ m/s.}$$

Substitute the value of V in eqn (i)

$$0.03 \, u_1 = 10.03 \, V$$

$$\therefore \quad u_1 = \frac{10.03 \times 1.448}{0.03} = 484.11 \text{ m/s}$$

∴ velocity of the bullet before impact, $u_1 = 484.11$ m/s.

7 *A pulley with two loads, connected by an inextensible cord is shown in fig. 22.7. If the load B moves downward with an initial velocity of 1.5 m/s, and uniform acceleration of 0.75 m/s², determine*

(i) Number of revolutions executed by the pulley in 2 sec.

(ii) Velocity and position of the load A after 2 sec.

Solution :

Since the cable is inextensible, the velocity and acceleration of C, a point on the rim of the outerpulley is equal to the velocity and acceleration of load B.

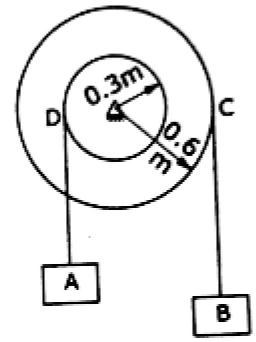


Fig. 22.7

given, initial linear velocity of load B, $(u_B)_o = 1.5$ m/s

initial linear acceleration of load B, $(a_B)_o = 0.75$ m/s²

for the load at outerpulley, radius = 0.6m

using the equation, $v = r\omega$

initial angular velocity of C, $(\omega_C)_o = \frac{(u_C)_o}{r} = \frac{1.5}{0.6}$

$= 2.5$ rad/s

using the equation, $a = r\alpha$

initial angular acceleration at C, $(\alpha_C)_o = \frac{(a_C)_o}{r} = \frac{0.75}{0.6} = 1.25$ rad/s²

(i) Number of revolutions of the pulley in 2 sec

using the equation, $\theta = \omega_o t + \frac{1}{2}\alpha t^2$

(t = 2 sec) $= (2.5 \times 2) + (\frac{1}{2} \times 1.25 \times 2^2)$

$\omega_o = 2.5$ rad/s $= 7.5$ rad

Converting 7.5 rad into number of revolutions,

Number of revolutions $= 7.5 \times \left(\frac{1}{2\pi}\right) = 1.194$ **(Ans)**

(ii) Velocity and Position of load A after 2 sec

Velocity and acceleration of the point D, on the rim of inner pulley is equal to the velocity and acceleration of the load A.

using the equation, $\omega = \omega_o + \alpha t$ $\omega_o = 2.5$ rad/s

$= 2.5 + (1.25 \times 2)$ $\alpha = 1.25$ rad/s^2

$= 5$ rad/s $t = 2$ sec

linear velocity of load A, $V_A = r_A \, \omega_A$ ($r_A = 0.3$ m)

$= 0.3 \times 5$

$= 1.5$ m/s

$\therefore$ displacement of load A, $S_A = r\theta = 0.3 \times 7.5 =$ **2.25 m**   **(Ans)**

OR

8 *A bar AB of length 1.2 m slides in xy plane as shown in fig. 22.20. The velocity of the point A is 5 m/s towards right. Determine*
i) the angular velocity of the bar
ii) the velocity of the end B, and
iii) the velocity of the mid point of the bar at the instant when the axis of the bar makes an angle of 30° with the horizontal.

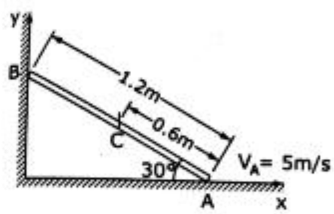


Fig. 22.20.

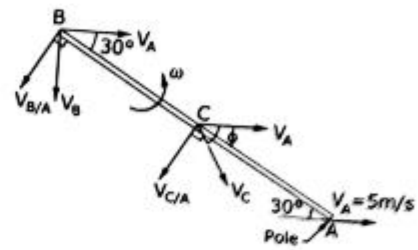


Fig. 22.21

Solution :

It is given that, the end A moves horizontally towards right. Hence the point A is taken as 'Pole' and the velocity component at B and C are shown in fig. 22.21.

The end B slides downwards when the end A moves towards right. Hence, V_A is taken towards right and V_B is taken downwards (along y axis) and the relative velocities $V_{B/A}$ and $V_{C/A}$ are at right angles to the axis of the bar. As velocity of A is known, the point A is taken as pole.

It is to be noted that, $V_{C/A} \neq V_{B/A}$, since, the position vectors of the points C and B are not equal with respect to the pole A.

(i) Velocity at B

Velocity at A, $V_A = 5$ m/s

Velocity of the end B, $\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$

or $\vec{V}_B = \vec{V}_A + r\,\omega$

or $\vec{V}_B = \vec{V}_A + l\,\omega \qquad (\because r = l)$

where ω is the angular velocity of the bar. The vector diagram of velocity component at the end B is shown in fig. 22.22.

Applying sine rule

$$\frac{V_A}{\sin 30} = \frac{V_{B/A}}{\sin 90} = \frac{V_B}{\sin 60}$$

or
$$\frac{5}{\sin 30} = \frac{V_{B/A}}{\sin 90} = \frac{V_B}{\sin 60}$$

solving,
$$V_{B/A} = \frac{5 \sin 90}{\sin 30} = \mathbf{10\ m/s} \quad \text{and}$$

$$V_B = \frac{5 \sin 60}{\sin 30} = \mathbf{8.66\ m/s\ (Ans)}$$

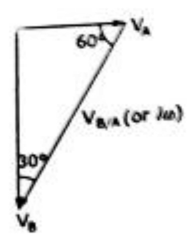


Fig. 22.22

To Check :

After finding $V_{B/A}$, the velocity at B, V_B can be checked by applying parallelogram law of forces. (Refer fig. 22.23)

$$V_B = \sqrt{(V_A)^2 + (V_{B/A})^2 + (2\,V_A \cdot V_{B/A} \cdot \cos 120)}$$

$$= \sqrt{5^2 + 10^2 + (2 \times 5 \times 10 \times \cos 120)}$$

$$= 8.66 \text{ m/s} \quad \text{(o.k)}$$

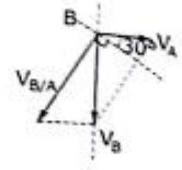


Fig. 22.23

(ii) Angular velocity of the bar

We know $V_{B/A} = 10$ m/s

but $V_{B/A} = l\omega$

$\therefore 10 = l\omega$

or $10 = 1.2\,\omega \quad (\because l = 1.2 \text{ m})$

$\therefore \omega = \dfrac{10}{1.2} = 8.333$ **rad/s (Ans) (anticlockwise)**

(iii) Velocity at mid-point C

Note that, the resultant velocity at C, V_C is not directed downwards. Let it be at an angle ϕ with horizontal as shown in fig. 22.24.

The vector diagram of velocity components at centre C is shown in fig. 22.24.

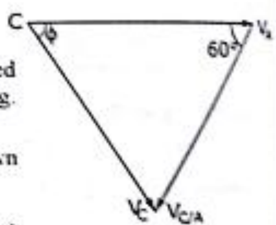


Fig. 22.24

we know $V_{CA} = r\omega \qquad$ (For the point C, $r = \frac{l}{2} = 0.6$ m)

$$= 0.6\,\omega = 0.6 \times 8.333 = 5 \text{ m/s}$$

To find ϕ

In fig. 22.24, $V_A = 5$ m/s ; $V_{C/A} = 5$ m/s.

$$V_C \cos\phi + V_{C/A} \cos 60 = V_A$$

or $\quad V_C \cos\phi = 5 - 5\cos 60 = 2.5 \qquad \text{..... (1)}$

$\quad V_C \sin\phi = V_{C/A} \sin 60 = 5 \sin 60 = 4.33 \qquad \text{..... (2)}$

$$\frac{\text{Eqn. (2)}}{\text{Eqn. (1)}} \Rightarrow \frac{V_C \sin\phi}{V_C \cos\phi} = \frac{4.33}{2.5}$$

or $\tan\phi = 1.732$

$$\therefore \phi = \tan^{-1}(1.732) = 60°$$

ie., the angle of resultant velocity at C, V_C with horizontal is 60°.

Hence, applying sine rule $\dfrac{V_C}{\sin 60} = \dfrac{V_{C/A}}{\sin \phi}$

or $\dfrac{V_C}{\sin 60} = \dfrac{5}{\sin 60}$ $\therefore$ V_C = **5 m/s. (Ans)**

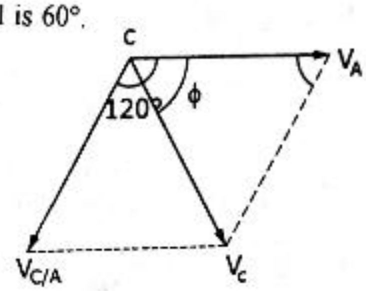


Fig. 22.25

To check

The velocity at C, V_C can also be determined by applying parallelogram law of forces at C. (Refer fig. 22.25)

$$V_C = \sqrt{V_A^2 + (V_{C/A})^2 + (2 \cdot V_A \cdot V_{C/A} \cdot \cos 120)}$$

$$= \sqrt{5^2 + 5^2 + (2 \times 5 \times 5 \times \cos 120)} = 5 \text{ m/s.} \qquad \text{(o.k)}$$